# Coordination in Scaling Actor Constellations
## The Advantages of Small–World Networks

Christian W.G. Lasarczyk[1] and Thomas Kron[2]

[1] Computer Science, University of Dortmund, christian.lasarczyk@udo.edu
[2] Institute for Sociology, Open University of Hagen, thomas.kron@fernuni-hagen.de

**Abstract.** The emergence of order in systems with many actors or agents is an interesting problem for sociology as well as for computer science. Both disciplines can contribute equally to its examination. In this article sociology provides a solution for "situation of double contingency" referring to Niklas Luhmann's theory of autopoietical systems. This is a coordination problem in social systems. First of all computer science can contribute techniques from the field of simulation. With these techniques it is possible to examine current as well as non–existing or no longer existing environments. Observations of the latter make it possible to draw further conclusions on the importance of the currently existing environment. At last computer science can utilise this knowledge about social processes especially in the domain of multi–agent systems.
Starting the from sociological theory of the dyadic "situation of double contingency" as mentioned above, our main focus is on large actor populations and their capability to produce order depending on different actors' constellations. Based on the theory for dyadic actor constellations we present our model of the actor. We do not want the actors to identify one another, so we do not need to modify this model if we scale up population size next and introduce constellations. Thereby we take regular, random and small–world constellations into account. After describing our measures of order we study emergence of order in different constellations for varying population sizes. By means of simulation experiments we show that systems with small–worlds exhibit highest order on large populations which gently decreases on increasing population sizes.

## 1 The Production of Social Order as a Coordination Problem

The explanation of how social order is generated, stabilised, and eventually changed by itself, is a main topic of sociology. The cause can probably be seen in the "annoying fact of society" (Dahrendorf), that humans have to deal with each other and from this social situations just develop. The reason for this relies in a parametric distribution of control and interests at certain resources, which forces the actors into one–sided or mutual dependencies. The actors are forced to process and accomplish their intention interferences [1].

The structural connection as the background of social acting [2] — the connection over mutual control of interesting resources — can be modelled by three basic types of social, strategic situations (co–ordination, dilemma, conflict). The *co–ordination problem* consists of the fact that the actors must find a tuning, which makes it possible, for all involved actors, to receive the possible utility. The interests of the actors converge here. For example, if some actors like to meet, but they do not know yet, in which place. If the individual and collective interests differ, then there is a *dilemma*. Who cleans the dwelling today, you or I? However, under certain conditions there are still cooperative solutions. This is no longer the case within a *conflict*, when the individual interests come apart completely. You always want to see soap operas whereas I want to see sports. This all has been examined thoroughly by sociology, and a few proposals have been made to solve this problem: social order is generated by a powerful state, the Leviathan [3]; by an "invisible hand" [4]; by norms [5], which are legitimated by values located in a cultural system of a society [6, 7]; or by rational action choices in consideration of a long common future [8].

In this contribution we just want to deal with the coordination problem, and within this problem class we deal with a specific problem that has to be solved: the difficulty of producing social order by solving the co–ordination problem *within scaling actor constellations*[3].

To repeat: the coordination problem is the simplest problem of the formation of social order. Hence many sociologists think that this problem has been investigated in all its problem dimensions. Particularly the rational choice theory assumes that dilemmas and conflicts are more interesting fields of scientific activity than coordination problems. Our suspicion is that simulation experiments open up new vistas which are ignored otherwise because there is simply a lack of the respective "analysis tool".

### 1.1 The Problem of Scaling in Coordination

The problem of scaling is an old issue in sociology. Already the German sociologist George Simmel has devoted the second chapter of his famous "Sociology" of 1908 to the "quantitative definiteness of the group". There he emphasised that on the one hand threshold levels of a group size just make certain social formation possible at all. On the other hand an increasing group size can make realisation of such formations more difficult. As an example he refers to a specific problem of social order: "So one can e.g. ascertain that total or approximated socialistic orders always have been accomplishable in small circles, but always have been abortive in great ones"[9]. In fact, Simmel has analysed the formal consequences of the scaling of the group size less than the influence on the relation of society — personality (individuality). Nevertheless we can find arguments in his scripts for the relevance of the *Zahlbestimmheit* in the arrangement of the group in subgroups, whereby (local) independency and mobility on the one hand and on the other hand (global) coherence are possible at the same time (one speaks of "glocalisation"). Those were not only first clues for the sociological concept of

[3] So, in this article we are just considering one of the two relevant scaling dimensions (see Schimank in this anthology).

“social differentiation”[10], but also, as we will see, first precursors for the model of small–world networks.

To point out the difficulties with scalings on the co–ordination problem we will take a game–theoretical view. Game–theoretically formulated we have a commonness of the interests in a succeeding co–operation with a missing dominant strategy, and the existence of several equilibriums as well as a (pareto-) optimum of a once found solution.

| | | B: 1 | B: 2 |
|---|---|---|---|
| A | 1 | 4,4 | 0 |
| A | 2 | 0 | 4,4 |

**Table 1.** Game–Theoretical Modeling of the Co–Ordination Problem

Without reference points the actors can build mutual action forecasts in an infinite recourse without arriving at a result, particularly if the number of action alternatives is high. In small communities where everybody can observe the other’s actions, the actors will be able to find a solution in a while by trial and error, or they can talk with each other and find an all–side accepted “focal–point”. But this will not be possible if the actor constellation exists of a such a great number of actors that the conditions of mutual observability and suggestibility as well as the dependency of the actor on the success of the cooperation is no longer given. Then at least[4] the coordination problem reemerges.

### 1.2 Double Contingency

The absence of the important starting point as the main difficulty of the co–ordination problem within the emergence of order is known in sociology as the “problem of double contingency”. Talcott Parsons [11], has formulated this problem as follows[5] : “The crucial reference points for analysing interaction are two: (1) Each actor is both acting agent and object of orientation both to himself and to the others; and (2) that, while acting, the agent orients to himself and to others, in all primary modes of aspects. The actor is knower

[4] Furthermore, there could be a qualitative step from the coordination problem to a dilemma if one assumes that there are only *rational* actors. Then the scaling means that everybody thinks of the own cost–value–ratio if he participates in solving the problem: the costs are for sure, but the own contribution to the solution is getting lower the more actors are involved. And if one will decide to participate nevertheless, how can he be sure that the other will do so, too? The result is, that nobody will participate but waits for a free–riding possibility.

[5] In an earlier version, Parsons’ [12] solution for the problem of double contingency had a much more economical bias. See also Münch [13].

and object of cognition, utiliser of instrumental means and a means himself, emotionally attached to others and an object of attachment, evaluator and object of evaluation, interpreter of symbols and himself a symbol." According to Parsons, Niklas Luhmann[14] identified the problem of double contingency as the main problem of producing social order. The problematic situation is this: two entities[6] meet each other. How should they act, if they want to solve the problem of contingency, that is, if necessities and impossibilities are excluded?[7]

Luhmann's assumptions for the solution of the problem of double contingency refer to self–organisation processes in the dimension of time. In a first step an actor begins to act tentatively, e.g., with a glance or a gesture. Subsequent steps referring to this first step are contingency reducing activities, so that the entities are enabled to build up expectations. As a consequence, a system history develops. Beginning from this starting point further mechanisms could be instituted to generate order, such as confidence or symbolic generalised media.[8] Thus in this perspective, social structures, social order, or social systems are first of all structures of mutual expectations. That is, every actor expects that the other actor has expectations about its next activity. In this paper we act on the assumption of the situation of double contingency as the origin of social order referring to co–ordination problems[9].

Summarised, the solution of the problem of double contingency presupposes at least the motivation of the actors by expectation–certainty as well as their possibilities of forming expectations over expectations. Accordingly we model our simulation scenario, we now want to describe briefly.

## 2 Modelling the Situation of Double Contingency

The basis model of the simulation scenario consists of agents, able to mutually signal themselves $N$ different symbols. The same number of $|N|$ different symbols is available for each agent and determines the scope of action and thus the

[6] The term "entity" denotes what Luhmann[14] called "Ego" and "Alter", and Parsons called "actor".

[7] One of Luhmann's basic assumptions is that both actors are interested in solving this problem. Luhmann[14]: "No social system can get off the ground, if the one who begins with communication, cannot know or would not be interested in whether his partner reacts positively or negatively." But the question remains: Where does the motivation (interest) come from? According to Luhmann, an answer should not consider actor characteristics (like intentions) as starting point for system theory. We think that Luhmann falls back to his earlier anthropological position (see Schimank [15, 16]) and assumes a basic necessity of "expectation–certainty", that is, that Alter and Ego want to know what is going on in this situation. A fundamental uncertainty still remains and takes further effect in the emerged systems as an autocatalytic factor. See also the approach to formulate "double contingency" from the perspective of a communication network as provided by Leydesdorff [17].

[8] For new simulation experiments about the genesis of symbolic generalised media, see Papendick/Wellner [18].

[9] We have done this before (see [19–21]).

contingency. These symbols are sent successively, individually, and alternately. There is no predisposed relationship or metrics between the symbols, represented as numbers. In the course of a simulation relations can develop by the way agents use the symbols. Two agents, chosen from the entire population, transmit in turn a symbol to each other[10] , so a situation of mutual observation exists. We take each symbol by an action, whereas each action is represented by a symbol one–to–one.

### 2.1 Action Motivation

Which motivations do the agents have for the selection of the symbols? According to the sociological analysis of the problem of double contingent situations explained above, we assume only two basal motivations[11]:

- Expectation–certainty, i.e., the agents want to predict the reactions of the other agents to own activities as well as possible. In other words, the agents want, that their expectations will not become disappointed by the reactions of other agents.
- Expectation–expectation, i.e., the agents want to accomplish the expectations of the other agents as well as possible.

### 2.2 Memory

The memory serves as a storage of action/reaction–combinations in the past. From this information the agents compute expectations to the future. We use a square matrix $X$ as the agent memory, which is stretched by the quantity of possible activities and reactions. All values of the matrix are initialised with a very small positive value

$$0 < x^{\text{init}}_{\text{action,reaction}} \ll 1 \ . \tag{1}$$

The agent learns a reaction following an activity by raising the according value within the matrix by one

$$x^{\text{new}}_{\text{action,reaction}} = x^{\text{old}}_{\text{action,reaction}} + 1 \ . \tag{2}$$

The value of a matrix entry rises at the rate the appropriate action/reaction–combination occurs in interactions of the agent.

[10] Because the agents do not differentiate explicitly between information and message, we do not model Luhmann's communication term, which consists of a three–way selection from information, message, and understanding.

[11] Further possible motivations, e.g. an interest in possible resources, remain unconsidered in the model. We particularly follow Luhmann, who considers intentions as too sophisticated for modelling the situation of double contingency: the pursuit of the own use is a much to fastidious attitude, than one could generally presuppose it [14].

Learning is associated with forgetting. For this reason, there is the possibility of selecting a value $r_{\text{forget}}$ (forgetting rate), which is added after learning to each matrix entry

$$x_{i,j}^{\text{new}} = x_{i,j}^{\text{old}} + r_{\text{forget}}, \quad \forall x_{i,j} \in X \ . \tag{3}$$

So the value $r_{\text{forget}}$ determines the rate at which the matrix entries assimilate.[12]

### 2.3 Choosing an action

Starting point of choosing an action $a$ is the last action $b$ of the other agent[13]. So you can always interpret an action as a reaction, which is performed by the agent in the following steps:

1. Calculate for each action a the action value $AV_b(a)$ as a combination of expectation–certainty and expectation–expectation.
2. Select a reaction on the basis of these action values.
3. (Re-)Act and if necessary[14] store the reaction.

We will explain these steps now.

**Calculation of the Expectation–Certainty (*EC*).** As already suggested, the expectation–certainty corresponds to the desire of being able to estimate the reaction of the interaction partner. For this it is important, that the other agents reacted unambiguously to a symbol in the past. If each possible reaction takes place with same probability, then the consequences of an activity are not foreseeable. As a measurement for expectation–certainty we take the so called Shannon–entropy [22] from the field of information theory. In order to be able to determine the expectation–certainty for an activity $a$ we take the vector $\boldsymbol{x_a}$ from our memory matrix $X$. This stores the frequency of all reactions plus the added forgetting constant $r_{\text{forget}}$. We normalise this vector and interpret its entries as probabilities for the possible reactions to activity $a$. So, the expectation–certainty for activity $a$ is computed as

$$EC(a) = 1 + \sum_{i \in \boldsymbol{x}_a} i \log_{|N|} i \quad . \tag{4}$$

This value is independent from action $b$ the agent has to react to.

**Calculation of the Expectation–Expectation (*EE*).** By the inclusion of expectation–expectation agent $A$ considers the own desire for certainty (as expectation) as well as the expectation of the other actors, agent $A$ is interacting with.

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[12] Without becoming equal as a result of this increase.
[13] The agents store their last action and send this one first if they meet another agent. At the beginning this action is chosen randomly.
[14] The own activity as reaction is only stored explicitly if the Ego–memory is used.

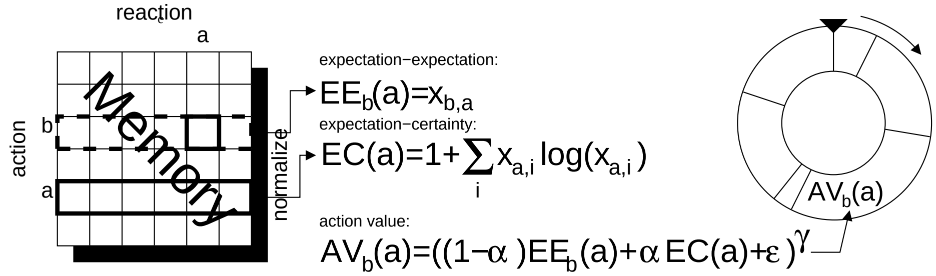


**Fig. 1.** This figure shows schematically the steps necessary to set the probability with which action $a$ is chosen as reaction to symbol $b$.

Also this calculation is based on the agent's memory, in which agent $A$ stores the reaction of the other agents to agent $A$'s action. This leads to agent $A$'s expectation, that the other agents expect the same reaction agent $A$ expects from them[15].

Starting from the action $b$ of the interaction partner the associated vector $\boldsymbol{x_b}$ gets normalised. Its entries $x_{b,a}$ are interpreted as the probability the interaction partner expects activity $a$:

$$EE_b(a) = x_{b,a} \quad . \tag{5}$$

**Combination of Expectation–Certainty and Expectation–Expectation.** The computed expectation–certainty $EC$ and expectation–expectation $EE$ for every possible action $a$ now have to be combined to an action value $AV$. To which parts the single values enter the action value, is determined by a factor $\alpha \in [0, 1]$

$$\mathrm{AV}_b(a) = (1-\alpha)\cdot \mathrm{EE}_b(a) + \alpha \cdot \mathrm{ES}(a) + \epsilon \quad . \tag{6}$$

The small value $\epsilon$ is added in order to ensure that no action value becomes zero. If an action value becomes zero, this action won't be taken into consideration. Therefore we construct our model in a way that all symbols are always possible alternatives for the action selection. This corresponds to latent uncertainty of the agents mentioned above as an autocatalytic factor.

**Action Selection.** Before the actual selection of the activity is done, all action values again will be taken to the power of $\gamma$ and then be normalised. This proceeding makes a continuous transition possible between the random selection of actions ($\gamma = 0$), the proportional selection ($\gamma = 1$), and the maximising selection [16] ($\gamma \gg 1$). Finally, the action is selected proportionally to the action

[15] If the computation is done on the Ego–memory (instead of the so called Alter–memory), which only serves for the storage of own reactions to other agents actions, then the agent acts in the same way, as it already did in its past as a reaction to the activities of the others.

[16] For the relevance of the logic of selection for a sociological explanation see Esser [23, 24].

value exponentiated with $\gamma$. Figure 1 shows a summary of the single steps to the activity choice.

### 2.4 Observers

In our basic simulation model two agents interact with one another, who are selected randomly from the quantity of all agents. In addition, we are able to annul the anonymity of the interaction by permitting observers. These take part in interactions in the sense of participation, but not actively. So observers learn from the behaviour of the other agents.

## 3 Modeling General Actor Constellations

We place the agents into parameterised small–world networks[17]. The scientific origin of small–world networks goes back to an experiment of Milgram [29], who had discovered that two arbitrary persons in this world are separated on the average only by six other humans[18]. The question is, how "six degrees of separation" are possible. The graph–theoretical formulation of the problem reads: How can one connect several billion of vertices with edges, so that starting from any point $A$, one can reach any point $B$ just by following the edges without more than six intermediate steps in average? The Hungarian mathematician Paul Erdös discovered, that independently of the number of points a relatively small percentage of coincidentally distributed connections (edges) are sufficient in order to get a completely connected graph. And the larger the number of vertices becomes, the more this percentage is reduced. The problem her is, that social relations in a social world are not random. Family and friends do not represent random graphs. Here Granovetter [30] points out that there are not only strong but weak relations too, which can have a strong influence (for instance for job procurement). Weak relations could build social bridges[19].

Thus Granovetter shows that weak social relations can produce social structures with properties similar to small–world structures (job offers by acquaintances lead to small characteristic path lengths, while a circle of friends leads to high clustering). So Granovetter owes an explanation of the mechanism to create such structures, too. But how can we reconstruct such networks?

[17] See [25–28], for an actual overview and further developments.

[18] Few years ago the German journal 'Die Zeit' had looked for the shortest connection of an Falafel–lunch–owner in Berlin with Marlon Brando. Not more than six intermediate steps were necessary. The New York Times repeated this play, that was called "Six Degrees of Monica" (Monica Lewinsky was meant) with the same result.

[19] Behind Granovetter's argument hides the picture of socialisation, which is characterised by strongly connected clusters, from which only few connections penetrate the cluster environment. This structure is an accumulation of complete graphs, in which each vertex is connected with each other vertex within the cluster, and in which only a few relations connect the different clusters. One can recognise the picture of society as an accumulation of autopoietic, structural coupled systems, too.

Here begins the work of Watts and Strogatz [31–33] , who have developed a model, which is suitable for the production of static small–world networks. We now present this model briefly and describe a little modification.

Dissatisfied with the fact that network topologies are modelled either as totally coincidental or as completely arranged (regularly)[20], while most biological, technical, and also social networks [34, 35] lie between these two extremes, Watts and Strogatz [31] have developed a model, which makes the interpolation between these two topologies possible. By doing so, structures develop with high clustering, comparably with the regular lattices, and with small characteristic path lengths, as can be found in random graphs. They call the developing structures small–world networks.

Starting point is a regular lattice with $n$ vertices arranged in a circle. Everyone of those vertices is connected by $k$ edges to $k/2$ vertices on the left and $k/2$ vertices on the right. This regular lattice is cyclically gone through $k/2$–times. First the edges to the direct circle neighbour of a side are rewritten with the probability p, i.e., the connection to the circle neighbour is solved and the regarded vertex is connected with any other vertex, it has not been connected to yet. In the following round the next circle neighbour is regarded and so on.

With this kind of the construction one receives the regular lattice for $p = 0$, for $p = 1$ a random graph. Watts and Strogatz are interested in the structural characteristics of nets with $0 < p < 1$, the range between order and randomness. Against $p$ they investigate the characteristic path length[21] $L(p)$ as a global characteristic of the nets and the cluster coefficient[22] $C(p)$ as a local characteristic. It turned out that for a small interval of $p$ nets occur, whose characteristic path length $L(p)$ is comparable with those by random graphs, while the cluster coefficient $C(p)$ corresponds still approximately to that of the regular lattice. That is a characteristic, which is provable in many biological, technical, and social nets and which allows a high speed for signal propagation and synchronisation in dynamic systems.

We are of the opinion that in a small population, where no local separation exists, every agent is able to meet every other agent. But nevertheless preferences exist, which lead to small–world like constellations. This contrasts to Watts' and Strogatz' static modelling of small–world networks. Once the edge rewriting procedure is finished, possible connections are fixed.

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20 Regular graphs are characterised by the fact that each vertex owns accurately the same number of edges. In contrast to this the edges are completely random within the random graph.

21 The average path length of the shortest path between two vertices is called characteristic path length.

22 The cluster coefficient is to quantitatively show the tendency for clustering. If $k_i$ is the degree of a vertex $i$ and $E_i \subseteq E$ is the set of the edges, which connect the vertices of its neighbourhood among each other, then its cluster coefficient amounts to $C_i = |E_i|/\binom{k_i}{2}$ . This coefficient reflects the relationship between existing and possible edges in the neighbourhood. The average of the coefficient of all vertices is the cluster coefficient of the graph.

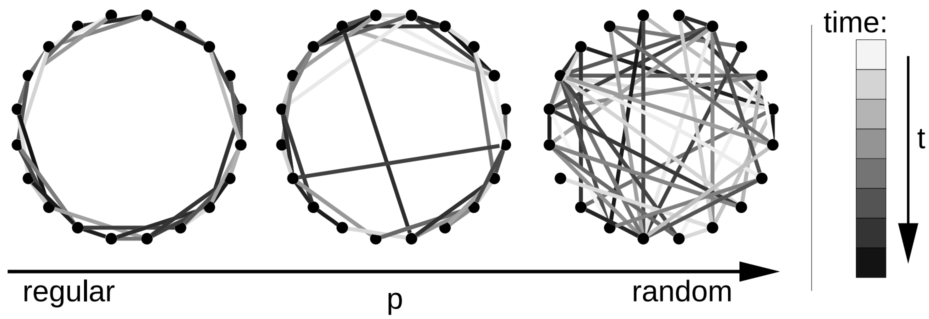


**Fig. 2.** Possible sequence of interactions depending on probability $p$ for edge rewriting.

For this reason we extended our model so that the interaction structure in one time period corresponds to a small–world network, however though each agent still has the possibility of interacting with every agent. On the basis of the original model by Watts/Strogatz we approximate that the probability $p(e_x y)$ for the existence of an edge between the vertices $x$ and $y$ is proportional to

$$p(e_{xy}) = \begin{cases} 1 - \beta \left(1 - \frac{p}{n-k)}\right)^{k-2d_{xy}+2} & , \quad d_{xy} \leq k/2 \\ 1 - \left(1 - \frac{p}{n-k)}\right)^{k} & , \quad \text{otherwise} \end{cases} . \tag{7}$$

The variables $p$, $k$ and $n$ have the same meaning as in the previous model. Within this model every agent can still interact with every other agent, but the probability to do so depends on its distance $d_{xy}$ on the circle. The first of the two interacting agents is selected randomly, the probability of the second agent is proportional to $p(e_{xy})$. Observers are selected proportionally to the sum of both $p(e_{xy})$ values. For random agent constellations we use a pseudo random number generator. Figure 2 shows possible interactions between the agents located on a circle during a longer period of time, exemplary.

## 4 Measures of Order

Before we present the results, we first explain our measure of order. It might have become clear that it concerns the achieved order, but how can order be measured? Sociology offers only few concrete references (for an overview see [36, 37]). According to these we suggest two measures[23], with which we measure the order achieved.

### 4.1 Systemic Integration

A rather macroscopic measure of order is *systemic integration*. It represents the certainty of the "average agent"

$$\overline{C}(b) = 1 + \sum_{\forall a \in N} \overline{AV_b}(a) \cdot \log_{|N|} \overline{AV_b}(a) \ . \tag{8}$$

reacting to a symbol $b$ weighted by the frequency $p(b)$ with which this symbol is used in the past. This leads to a systemic integration of

$$I = \sum_{\forall b \in N} \overline{C}(b) \cdot p(b) \ . \tag{9}$$

### 4.2 Weighted Systemic Integration

Obviously it is easy to achieve a high systemic integration in systems if the number of factually communicated symbols has been reduced to two or three after a while. To uprate highly integrated systems and a large number of symbols we weight the systemic integration with the number of communicated symbols in the preceding time interval. In other words: the *weighted systemic integration* of a system $A$ is higher than the *weighted systemic integration* of a system $B$ if both systems have the same systemic integration but system $A$ is able to cope with a higher contingency (larger number of symbols) at the same time.

## 5 Results

Table 2 shows the settings of the parameters described above.

We measure systemic order for different population sizes in the three described agent constellations (random, small–world network, regular). Starting with a population size of 64 agents we double the number of agents three times. We choose the simulation duration such that every agent in average actively takes part in 5000 interactions. This means that we simulate 160.000 steps for a population size of 64 agents, 320.000 steps for a population size of 128 agents, 640.000 steps for a population size of 256 agents, and we simulate 1.280.000 steps

[23] We have tested and used further measures in other places, e.g. reduction. We counted the number of different symbols, which were selected in a certain time interval by the agents. The smaller the number of selected symbols, the larger the achieved *reduction* of the agents, and the larger the order. This is a macroscopic order perspective. *Certainty* is a microscopic measure for the emergence of order measuring the certainty of the agents over actions selected by them. A high value represents high certainty and thus a high degree of order. To calculate certainty we use the entropy over all normalised action values of possible actions in reaction to symbol b.

**Table 2.** Parameter settings used in our simulation runs.

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| number of agents | 64–512 | number of symbols $\|N\|$ | 50 |
| weight $\alpha$ ($EE$–$EC$) | 0.5 | selection exponent $\gamma$ | 2 |
| avg. nr. of time steps per agent | 5.000 | interactions per step | 5 |
| neighbourhood $k$ | 6 | observer | 2 |
| forget rate $r_{\text{forget}}$ | $10^{-3}$ | | |
| $p_{\text{regular}} = 0$ | | $p_{\text{SWN}} = 0.1$ | |

for a population of 512 agents. We carried out 30 runs for each combination of population size and agent constellation and describe the average results. Figure 3 shows the dependency between the average weighted systemic order at the end of simulation and the number of agents for all three constellations.

As you can see, no order originates within large populations within a random constellation. Nevertheless, order rises for smaller populations sizes. The random constellation differs from all the others inasmuch as emerging order is *here* a time–consuming task. Figure 4, showing the time dependent emergence of order for different constellations with 128 agents, clarifies this fact. Agents within such a random constellation minimise the number of communicated symbols. This leads to a higher certainty in choosing a reaction because factually fewer symbols come into question. This simplifies the creation of order. High systemic order with many symbols is possible, too, and we have to rate this order differently than systemic order arising from a reduced number of symbols. For that reason we weight systemic order with the number of used symbols, as mentioned above.

Past research [38, 21] shows that emergence of order in random constellations is not only a time consuming task but also happens only if the system is not perturbed or perturbation is low. This condition is fulfilled here.

If you compare weighted systemic order of random and other constellations while scaling up agent population size, you see that order emerges in regular and small–world constellations; even though it decreases for larger populations. Thereby you find higher order within small–worlds in comparison to regular structures.

The main aggregation affect we concentrate on is the emergence of order in large agent populations for *some* constellations. We ascribe this to the vision range of the agents. While agents in regular constellations only interact with their neighbourhood, agents in small–world constellations primarily but not exclusively interact with their neighbourhood, whereas in random constellations agents choose their interaction partner arbitrarily.

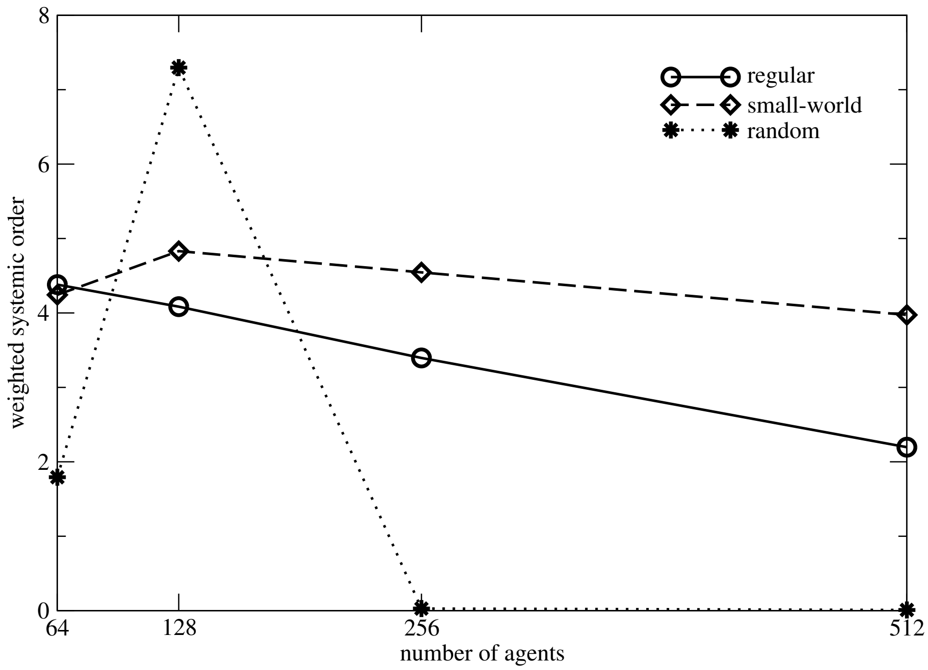


**Fig. 3.** Average systemic order of 30 independent runs for regular, random and small–world constellations and four different population sizes. We connect the measure points for optical reasons.

The agents have no possibility to shape individual expectations at all, so their expectations refer to a "generalised agent"[24] in populations with $N > 2$. The word "generalised" denotes, that the agents expect, the other agents react like the average of all agents they gained experience with before (through interaction or observation).

Expectations towards such a generalised interaction partner are build up in the memory of all agents. This happens to agents within a neighbourhood of regular or small–world networks on the base of comparable experiences, so their expectations equalise in the course of time. From a sociological point of view a mutual fulfilment of (expectation-) expectations evolves.

By increasing the size of the neighbourhood the process of adaptation is getting more difficult. Or to restate this sociologically: By increasing the size of "community", "collective consciousness" (*Kollektivbewusstsein*) gets lost. We think of "collective consciousness" as the ability to adapt expectations and (expectation-) expectations. Within random constellations there is no neighbourhood in the narrower sense because agents may interact with every other random agent. The

[24] The generalised agent is understood in the sense of a "generalised other" in terms of George Herbert Mead [39] as the sum of expectations of all, which are relevant in a certain situation.

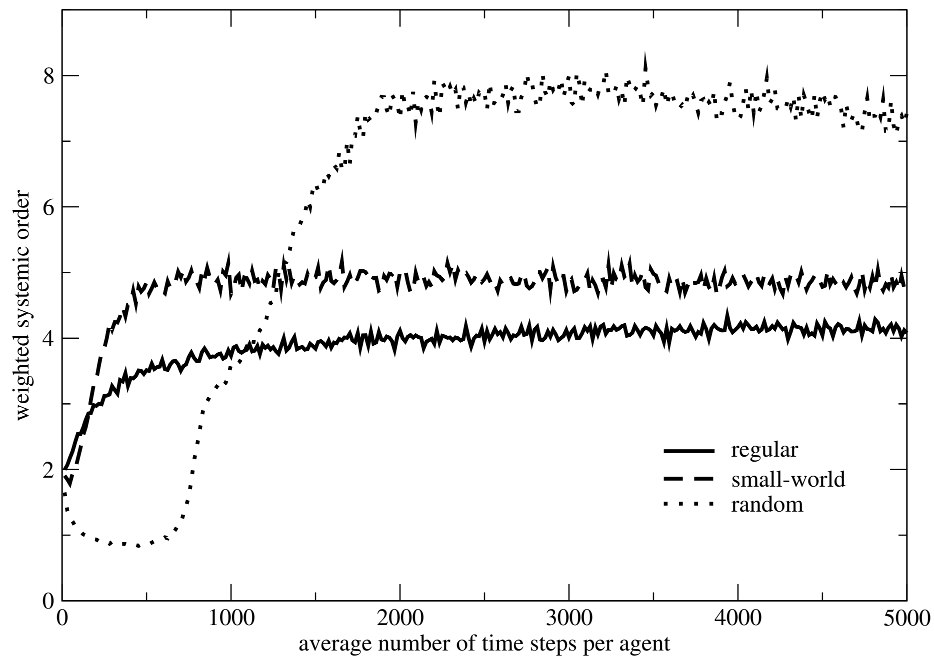


**Fig. 4.** Emergence of order in regular, small–world and random constellations with 128 agents. Albeit order in random constellations is heigh at the end, it is a time consuming task.

process of adaptation is difficult right from the beginning. Scaling up populations size makes it difficult to build up expectations by mutual adaptation. In random constellations all properties have global effects. For this reason populations in this kind of structure show a high degree of order if population size allows a gradual convergence, or they do not show any order at all.

As you can see further, with raising numbers of agents systems with regular constellations have a lower degree of weighted systemic order than systems with small–world networks. We call this effect *scaling resistance* of small–world agent constellations and attribute this effect to the small characteristic path length of small–world networks. Remember, that for systemic order the question of how certain an agent reacts to an action plays an important role. To restate this question: Does the agent know the expectations to its action and is it able to expect the reaction of its interaction partner?

Expectations evolve from interaction. Thereby, because of the concept of generalisation mentioned above, interactions with neighbours (or other previous interaction partners) of an agent could have been sufficient to built up expectations that correspond with the behaviour of the agent. In our small–world constellations, arbitrary agents are able to interact with each other. With a low probability these could be distant agents. So agents do not only build up expectations towards the interaction partner, but also towards their generalised neighbourhood. Therefore (after this interaction) the agent is able to live up the expectations of a distant part of the population better than before and furthermore these agents better live up to their expectations. The agent changes its behaviour and so carries these newly build up expectations into its own neighbourhood.

For sure, the adaptation of expectations also happens in regular agent constellations. But here two interacting agents have a big intersection within their past interaction partners. So just interactions with a small number of unknown agents (agents not belonging to the known neighbourhood) benefit from the adjustment of expectations.

While, metaphorically speaking, expectations towards behaviour at the "back of beyond" must be handed over step by step in regular constellations, agents in small–world constellations benefit from the facility to adapt their expectations towards distant parts of the population. Thus, in such structured systems a higher degree of systemic order is possible.

## 6 The Evolutionary Advantage of Small–World Networks for social systems

The thesis that increasing population size can lead to a change of social structures, is common in sociology for a long time. In 1893 Durkheim already saw the cause for the development from simple, segmentarily differentiated societies to complex, division of labour organised societies in a mechanism, that almost drives the actors to specialisation, so that these — as unintended consequence [40] — build up new social structures.

According to Durkheim this unintended aggregation effect is attributed to an increasing population in a limited area and the social density developing from it: "If the society comprehends more individuals which are in close contact at the same time, then the effect [of increasing division of labour] follows necessarily." [41]. Increased social density leads to increased competition. Specialisation by division of labour, so Durkheim referring to Darwin, decreases the competition pressure. Specialisation is the mechanism for occupying ecological niches of the social competition — at least if the demand for relevant resources is smaller at the same time than their offer, and there are no other possibilities to escape the competition. We do not want to examine the plausibility of Durkheim's threads here. We only want to state this as an indication that sociology has seen the increase of population in a quite prominent place as a cause for important changes of social structure within the framework of social development (social evolution).[25]

[25] It seems that Durkheim [41] implicitly thought of small–world networks while describing the social–structural changes. He said that the disbandment of segmentary society, which is characterised by high individuality and demarcation of social segments (thus high clustering), leads to the fact that an "interchange of movements between the parts of the social mass, which had not affected each other till then",

At this point, we want to ask the question, if small–world structures are the (inescapable?) result of an evolutionary process. Thereby we do not want to describe the single step of evolution, but rather concentrate on those forces, which, as a selection criteria, put pressure on communities and maybe contribute to the establishment of small–world structures. We take advantage of knowing the result of the evolutionary process — small–world structures. Results presented here and in early research let us conclude the reasons, that lead to the establishment of small–world networks.

First, we want to make some assumptions. Our starting point is a regular structure: a multitude of self–contained sets of individuals. Sociologically, we want to interpret them as communities like they occur in real world as families, tribes, or prides. We do not want to take into account how these closed communities arise, for sure another evolutionary process leads to them. In fact many higher life–forms live in such communities, e.g. lactation forces the instantaneous integration of all newborn mammals into a community (see also Kron [42]). We further assume that the territories of communities are not spatially separated, but individuals can meet individuals of other communities, even if they do not want to. In the end, we assume that the encounter of two individuals of different communities can have repercussions to their communities, if the meeting individuals do not have expectations of the behaviour of the other individual (see Münch [43] for this assumption as a starting point of social differentiation). Moreover, we assume in terms of methodological individualism, that each community is finally based on its members. Without extraneous causes — evolution shall be the only force here — the individuals on the one side produce certain characteristics of communities, i.e. certain social structures. On the other side, it's the community forming the action, which penalises its members — if the community is evolutionarily unfit for example. From this point of view, also evolution considers the "duality of action and structure" [44].

The previous assumptions suggest, why the regular starting structures do not endure. Expectation must not inevitably arise from the encounter of two individuals from different communities. Rather than this we can assume, that both individuals mutually guess "bad" motives (see [42, 45]), even though they are well disposed to the other. This situation can be harmless, but it also can lead to disadvantages for one or both of the interacting individuals. In this case individuals take advantage of making expectations out of their experiences

develops. Thereby the social system "generalises" itself, it will be, we would say today, more global. "The social relations [...] therefore becomes more numerous because they diffuse to all sides over their original borders. Thus the division of labour more and more progresses, the more individuals there are, who keep in touch sufficiently, in order to be able to interact". Durkheim also calls this "dynamical density". Finally, society organised by division of labour is the result of the scaling of the population and the increasing dynamic density: "The division of labour changes in direct relation to the volume and for the density of the societies; thus if it constantly progresses in the course of the social development, so because the societies became regularly more closely and more extensive in general" [41].

because these expectations can minimise the disadvantages (e.g., getting killed). It is sufficient if the sow turns tail and runs expecting to become eaten.

After getting clear that expectations can be an advantage, there are two possibilities how individuals can obtain those expectations. Firstly, individuals can form those expectations on the basis of *subjective* experiences, i.e. of their own life story. Or, secondly, those expectations can be imparted *collectively* within the community [26]. We assume here, that these expectations have to be made in lifetime because they are, e.g., too fugacious to emerge by evolution.

Let us look at the first possibility that every individual forms expectations based on subjective experiences. This possibility does not appear to be optimal because the primary situations, in which expectations can be formed, can involve the mentioned disadvantages. But even if such situations would always end positively for the participating individuals, it needs plenty of time until all individuals of all communities have formed their expectations. And in fact we have surveyed that the formation of order in random constellations is a longsome process. The situations get worse with increasing variety, which is equivalent to a large population in our model. The reason for the total breakdown in our simulation should be the restricted cognitive capacity of our agents, modelled by the forgetting. But even for *real* actors this is not an improper assumption (the authors speak from their own experiences).

The second possibility is, that every individual of a community forms expectations towards a few individuals outside of the community, and import these expectations to the own community. Thereby it is not important how these expectations are passed on, if by communication, or — as in our case — by mutual observation, so that it changes its behaviour because of adapted expectations. We have modelled this with the small–world constellation. This constellation shows construction efficiency in respect to (information propagation) performance and is the most economic constellation[46] tested here. Agents only occasionally interact with other agents outside of their neighbourhood/community. Their expectations, which are adapted by the interaction with other agents, are put forth in their own neighbourhood by a modified behaviour. We can observe that in this constellation there is a very rapid formation of order, and that this order is also approximately achieved in a scaling population. Furthermore, this constellation has been proved as very robust against interferences.

We interpret the fact, that expectation structures are able to form a high ordering first of all in those actor constellations, which are already identified in existing communities, as a first indication of their importance.

## 7 Conclusion and Outlook

Starting from the problem of double contingency following the "classical" definitions by Parsons and Luhmann, it is shown how social order emerges in scaling actor constellations. For this we augment the "simple" coordination–situation with

[26] This differentiation is purely analytical, whose components empirically should often be inseparably aligned.

the dimension of different actor constellations (random, regular/neighbourhood, small–world). The main result is that small–world networks evidently have a specific meaning in the formation phase of a social system. In the linear degeneration of small–world actor constellations, while scaling up the degree of actors, you can also see that these are less fragile than other constellations. Finally, we reason that small–world actor constellations are significant in the evolutionary process of social systems (especially if you take into account that small–world networks seem to be very resistant against interferences [38]) and have to be recognised in the explanation of the emergence of social order in scaling actor constellations.

We believe that small–world structures are not only in real world social systems of particular importance, but also in many other systems in which an appropriate ratio of robustness and adaptivity is needed to let a local convergence follow a global convergence.

Multi–agent systems seem to be predestined to profit from constellations similar to small–world networks because they are modelled on social systems based on division of labour[27]. They could take profit of small–worlds in such areas, where mutual adaption by communication and/or observation is needed, while the agents are intransparent otherwise. Adapting the communication system is a promising field here (see [47]).

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[27] But also in many other fields of computer science, where the influence of structure is just observed in its extreme value — regular and random — networks, could be interesting research areas for small–world networks.

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